Numerical Modeling and Comparison of Flash Thermographic Response

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Abstract. This paper discusses the numerical modelling aspect of the IR intensity response during the flash in flash thermographic analysis. Accurate modelling of flash response would be of help in characterizing the material/defect parameters in a flash thermographic non-destructive evaluation. The possibility of using log-normal distribution for modelling flash response is investigated and compared with two of the available models. The actual thermographic response is influenced by the type of flash tube/lamp, any filters used with the flash lamp and the hardware setup. In this study the flash response was recorded for a flash lamp with an IR filter. The normalized flash functions are compared using a finite difference solution of heat diffusion model. The challenges faced in implementing the lognormal distribution over the exponential decay are discussed. Finally, the use of log-normal distribution as a normalized shape function for the flash is verified using an experiment on a quartz specimen.

Introduction

Flash Thermographic Non-Destructive Testing (FTNDT) technique is a rapid non-contact NDT method used in a wide range of applications. FTNDT is an active thermographic technique where the temperature of the test surface is instantaneously raised with the aid of a flash lamp and the surface temperature is monitored using an infrared camera. The contrast images obtained at a specified frequency will be helpful in characterising the material properties and embedded defects of the test object.

Numerical models will be useful in FTNDT to determine critical equipment setting parameters such as the frequency and duration of data acquisition and to understand the experimental data obtained from samples that display unusual characteristics. In addition, where analytical solutions are not available or hard to obtain, numerical solutions would be of help. Sun [1] had created a numerical model using finite difference solution technique for layered material system including the volume heating and flash duration effects. Cielo [2] had shown that numerical model can be used to understand the distribution of fiber contents and subsurface delaminations. Balageas et al [3] had provided the first set of analytical solutions for layered materials with flash having different normalized shape functions. Their analytical solutions were found to be useful in understanding the influence of materials and flash functions for layered materials. Their analytical solutions also found to be useful in verifying numerical models created for FTNDT.

In this study a numerical model was created using finite difference solution technique. The model is verified with the available analytical solutions and used to further understand the flash duration effect. Flash duration effects were analysed using analytical solutions [3] for three different functions used to describe flash. Sun [4] had proposed an exponential decay function to describe the flash duration effect. In this study feasibility of using a log-normal distribution for a flash has been investigated. Even though the effect of flash duration is negligible at later times [3], it would be useful to have models to accurately describe the flash effect where the material characteristics to be extracted within the time where the flash duration effect is present.

Background

2.1 Theory

General heat diffusion equation can be given by

$$\nabla \cdot (K \cdot \nabla T) + \dot{q} = \rho c \frac{\partial T}{\partial t} \tag{1}$$

where $K, T, \dot{q}, \rho, c$ and t are thermal conductivity matrix, absolute temperature, body heat generation rate per volume, density, specific heat capacity and time respectively. In flash thermography the object inspected usually of plate like structure as shown in Figure 1. Therefore the heat diffusion can be assumed to be one dimensional. As a result the governing equation for flash thermographic analysis can be reduced to the following equation,

$$k \frac{d^2 T}{dz^2} + \dot{q} = \rho c \frac{dT}{dt} \tag{2}$$

In an ideal flash thermographic analysis, there will not be any internal heat generation. Therefore $\dot{q}$ can be assumed to be 0 and boundary condition can be appropriately assigned. In a general case however, volume heating could happen due to the flash, therefore the volume heating and the effect of flash duration can be included by utilizing the heat source term $\dot{q}$.
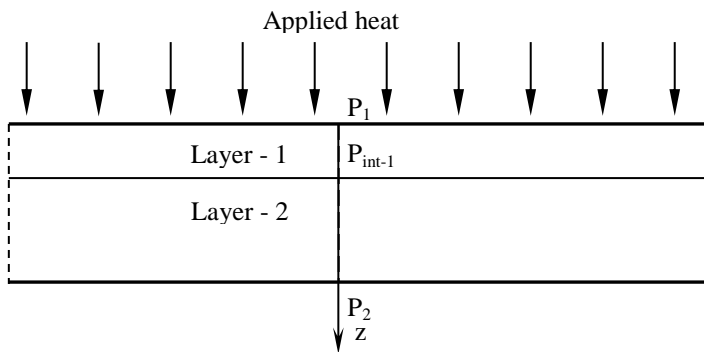

Figure 1: A two layered material system

Analytical solutions for a 2 layered material system, for a Dirac pulse of heat at the front surface, have been obtained by Balageas et al. [3] and is given by

$$T_d(t) = T_\infty \left\{ 1 + 2 \left[\frac{\sum_{i=1}^{2} x_i \omega_i}{\sum_{i=1}^{2} x_i} \right] \sum_{k=1}^{\infty} \frac{\sum_{i=1}^{2} x_i \cos(\omega_i \gamma_k)}{\sum_{i=1}^{2} x_i \, \omega_i \cos(\omega_i \gamma_k)} \exp\left(-\frac{\gamma_k^2 t}{\eta_2^2} \right) \right\} \tag{3}$$

where

T_∞ is the equilibrium temperature,
γ_k is the kth positive root of

$$\sum_{i=1}^{2} x_i \sin(\omega_i \gamma) = 0 \tag{4}$$

where,
$x_i = e_{12} - (-1)^i$
$\omega_i = \eta_{12} - (-1)^i$
$e_{12} = \dfrac{e_1}{e_2},$
$\eta_{12} = \dfrac{\eta_1}{\eta_2}$
with $e_i = \sqrt{k_i \rho_i c_i}$ and $\eta_i = \dfrac{L_i}{\sqrt{\alpha_i}}$

If the temperature evolution of a material system for a Dirac flash is known as $T_d(t)$, then for various flash described by $\varphi(t)$, the resultant temperature evolution will be given by

$$T(t) = \int_{t'=0}^{t} \varphi(t') T_d(t - t') dt' \tag{5}$$

In this study, three flash functions are considered. They are
    1. A flash model derived by Larson and Koyama [5]
    2. Exponential decay function proposed by Sun [4]
    3. Lognormal distribution function

Flash duration effects were considered by Balageas [3] for three different normalized shape functions. They are square function, triangular function and a function proposed by Larson and Koyama. Equation (6), (7) and (8) represent the shape functions to describe the flash effect given by Larson and Koyama, Sun and lognormal functions respectively.

$$\varphi(t) = \frac{t}{t_m^2} e^{-\frac{t}{t_m}} \tag{6}$$

$$\varphi(t) = \frac{2}{\tau} e^{-\frac{2t}{\tau}} \tag{7}$$

$$\varphi(t) = \frac{1}{S\sqrt{2\pi t}} e^{-\frac{(\ln(t) - M)^2}{2S^2}} \tag{8}$$

where t_m is the peak time of the flash, τ is the time constant, S and M are parameters describing the log-normal distribution. The closed form analytical solution for a single layer material, with the exponential decay flash, can be given by [4],

$$T(t) = \frac{Q}{\rho C L}\left[1 - e^{-\frac{2t}{\tau}} + 2\sum_{n=1}^{\infty}(-1)^n \left(\frac{n^2\pi^2\tau\alpha}{2L^2} - 1\right)^{-1}\left(e^{-\frac{2t}{\tau}} - e^{\frac{n^2\pi^2\alpha}{L^2}t}\right)\right] \tag{9}$$

In flash thermography Shepard et al [6] shown that the first and second logarithmic derivatives (1d and 2d) are also found to be effective in defect characterization. In this study the derivatives found to be helpful in characterizing the interface between the layers. The first and second derivatives are given by

$$1d = \frac{d[\log(\Delta T)]}{d[\log(t)]} \tag{10}$$

and

$$2d = \frac{d^2[\log(\Delta T)]}{d[\log(t)]^2} \tag{11}$$

2.2 Numerical Modelling

In Finite Difference solution methods heat equations are solved using Crank-Nicholson algorithm (see for example [7]). In summary the technique is to find solution (temperature) at certain time step using the known temperature of the previous time step. The vertical line P_1P_2 in Figure 1 is discretized and represented in Figure 2. At the beginning all the nodes shown in Figure 2 will be at a known initial temperature.
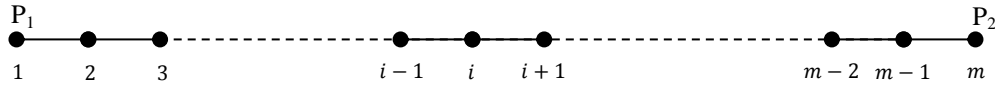
Figure 2: Spatial discretization of the layered system shown in Figure 1 along the depth

A general discretized model of the equation (2) can be obtained by considering the six nodes $(i-1)$, i, $(i+1)$ of j and $j+1$ time steps as shown in Figure 3.
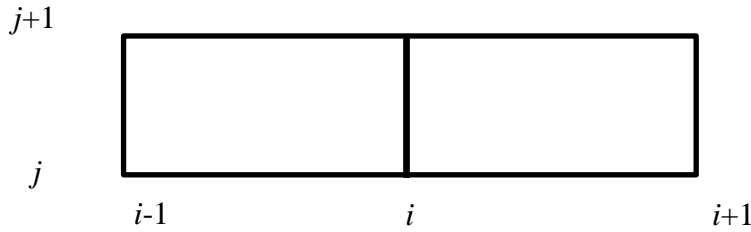
Figure 3: A Generalized grid for Crank-Nicholson algorithm

Equation (2) can be discretized using Crank-Nicholson method and is given by

$$k\left\{\frac{\left(T_{i+1}^{j+1} - 2T_i^{j+1} + T_{i-1}^{j+1}\right) + \left(T_{i+1}^j - 2T_i^j + T_{i-1}^j\right)}{2\Delta z^2}\right\} + Q_{ij} = \rho c \frac{\left(T_i^{j+1} - T_i^j\right)}{\Delta t} \tag{12}$$

where Q_{ij} represents the volume heating and flash duration effects. The above equation can be re-arranged as follows

$$-\lambda T_{i-1}^{j+1} + 2(1 + \lambda)T_i^{j+1} - \lambda T_{i+1}^{j+1} = \lambda T_{i-1}^j - (2\lambda - 2)T_i^j + \lambda T_{i+1}^j + Q_{ij} \tag{13}$$

where $\lambda = \frac{\alpha \Delta t}{\Delta x^2}$ and $\alpha = \frac{k}{\rho c}$

The above equation can be formed for nodes for a time step of $(j+1)$, given in Figure 2, starting from 2 to m-1, which results in a total of m-2 number of equations, while the unknowns are of m numbers. Therefore to obtain two more equations top (front wall) and bottom (back wall) boundary conditions are used. At the top surface if there is an additional heat flux (q_a) applied, the boundary condition can be given by

$$T_{j-1}^n = T_j^n + \frac{\Delta z q_a}{k_a} \tag{14}$$

Substituting equation (14) in (13) will result in a discretized equation to represent the top surface as

$$(2 + \lambda)T_i^{j+1} - \lambda T_{i+1}^{j+1} = (2 - \lambda)T_i^j + \lambda T_{i+1}^j + \frac{2\lambda\Delta z q_a}{k} + Q_{ij} \tag{15}$$

Similarly at the bottom surface for an additional applied heat flux of (q_b) the boundary condition is given by

$$T_{j+1}^n = T_j^n + \frac{\Delta z q_b}{k_b} \tag{16}$$

In the present study q_a and q_b are 0, as at the top surface the necessary heat diffusion have already been included in the general equation and the bottom surface is considered to be adiabatic. Substituting (16) in (13), discretized equation to represent the back-wall can be obtained as

$$-\lambda T_{i-1}^{j+1} + (2 + \lambda)T_i^{j+1} = \lambda T_{i-1}^j + (2 - \lambda)T_i^j + \frac{2\lambda\Delta z q_b}{k} + Q_{ij} \tag{17}$$

In case of multilayer materials the discretized equation at the interface between two materials, denoted by subscripts a and b, with negligible interface resistance can be given by

$$-\Delta_b k_a T_{i-1}^j + (\Delta_b k_a + \Delta_a k_b)T_i^j - \Delta_a k_b T_{i+1}^j = 0 \tag{18}$$

In this study spatial grid size $\Delta_a = \Delta_b = \Delta z$. In the discretized equation the heat generation can be given by

$$Q_{ij} = Q f_i \varphi_j \tag{19}$$

where Q is the amount of heat supplied, and in the past f(z) and g(t) are assumed to be exponential decay functions [1]. They are given by the following,

$$f_i = p\delta(z_i) + (1 - p)ae^{-az_i} \tag{20}$$

$$\varphi_j = \frac{2}{\tau}e^{-2\frac{t_j}{\tau}} \tag{21}$$

Where δ: Kroneker delta,
p —surface heat absorption factor varies from 0 to 1 and a is the attenuation coefficient to represent volume heating effect

Numerical model validation

3.1. Flash duration effect

The numerical model developed in this study have been validated for 2 sets of analytical results
1. The flash duration effect
2. Thermographic response of multilayer system

The flash duration effect on the numerical model has been compared by using the analytical solution given in equation (9). As shown in Figure 4, since the comparison was close enough for practical purposes, including other flash functions have been analysed using the numerical model. As described earlier, in this study 3 flash responses have been studied and they are

1. A function used by Larson and Koyama (L-K)
2. Exponential decay function used by Sun
3. A log-normal function

Figure 5 describes a sample of 3 flash functions used and the first derivative response for the corresponding flash in a single isotropic material. The parameters used in this analysis are $t_m = 1.667 \times 10^{-3}$ s, $\tau = 5 \times 10^{-3}$ s, $S = 0.6, M = -5.9$, $L = 2$ mm, $k = 1$ W/m/K, $\rho = 6050$ kg/m^3, $c = 413$ J/kg/K.
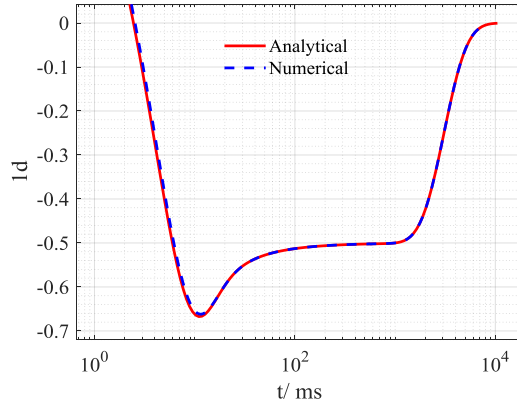

Figure 4: Comparison of first derivative plots obtained from analytical results and the numerical model
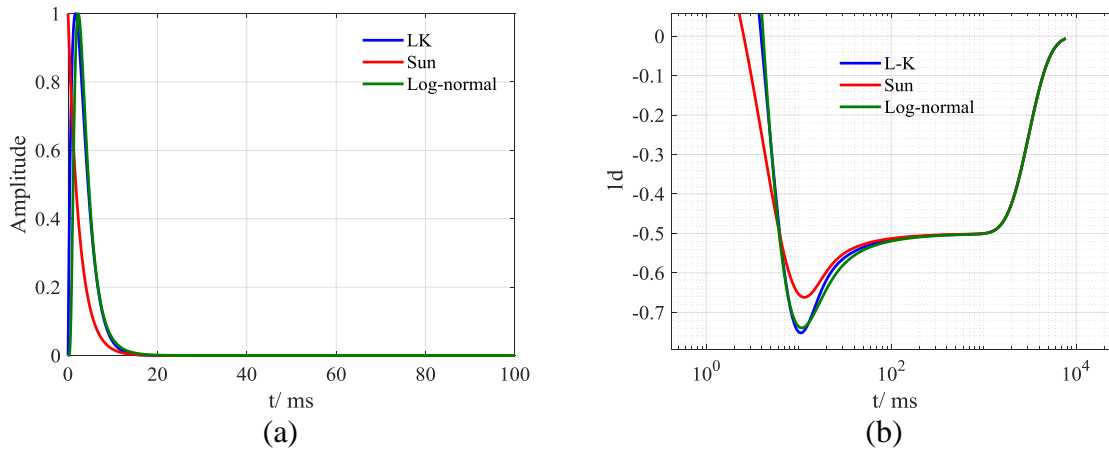

| (a) | (b) |

Figure 5: (a) Three different functions used to model the flash and (b) Numerical thermographic responses of the corresponding flash functions

3.2 Two-layer Materials System

The analytical solution for the temperature evolution on a two layer model given in equation (3) is compared with the numerical model for the parameters given in Table 1. The results are given in Figure 6.

Table 1: Material properties and thickness of the two layer system

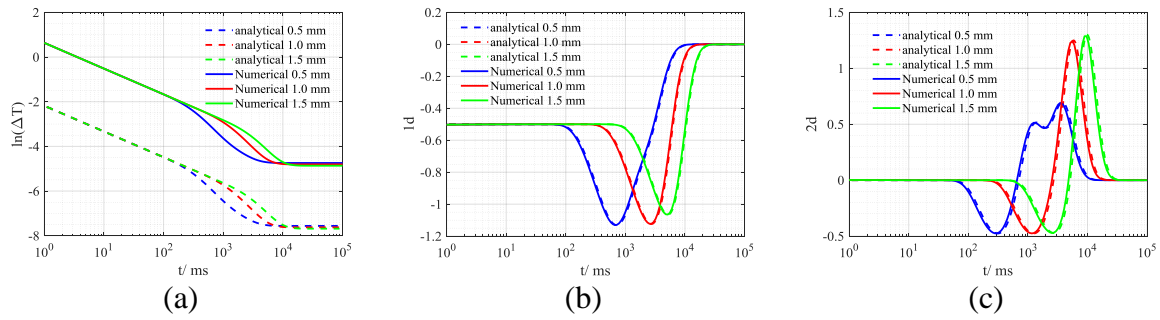| Properties	Top-Layer	Bottom-Layer
k / (Ton.mm/ s^3/K)	1	9
ρ / (Ton/mm^3)	6.05e-9	7.5e-9
c / (mm^2/s^2/K)	4.1322e8	4.8e8
α / (mm^2/s)	0.3824	2.5
Thickness/mm	L_1= 0.5,1,1.5	$L_2 = 5$

Figure 6: Comparison of Analytical Solution with Finite Difference Solution (a) temperature (b) 1d and (c) 2d evolutions for different thicknesses of the top layer

Results and Discussion

Since, the numerical model has been validated for known analytical solutions for flash duration effects and two layer materials system, the model has been used for further analysis using experimental data. Numerical model has the advantage of including any form of functions for flash and the volume heating. Especially where closed form solutions cannot be obtained or analytical solutions becomes complex for various flash functions or multi-layer material system. The flash thermographic results obtained for a quartz plate specimen of 5 mm thickness coated with Titanium Nitride have been used to compare the numerical results. Figure 7 describes the comparison of log-normal shape function with experimental thermographic data evolution. As seen in Figure 7, the flash duration effect is better compared with the second derivative plot than the first derivative plot. The modelling using exponential decay function neglects the rise time flash effects. Though the assumption will be good enough for most of the practical situations, for better accuracy flash model is to be improved. The use of log-normal function has the freedom of specifying the peak time and the width of the flash together as opposed to the normalized functions given by equations(6) and (7). However, the parameters S and M in log normal distribution have to be determined by trial and error method for a certain system of flash unit. In this case $S = 0.45$, and $M = 5.5$. It is also possible to modify the function given by Larson and Koyama by including a parameter to describe the width of the flash.
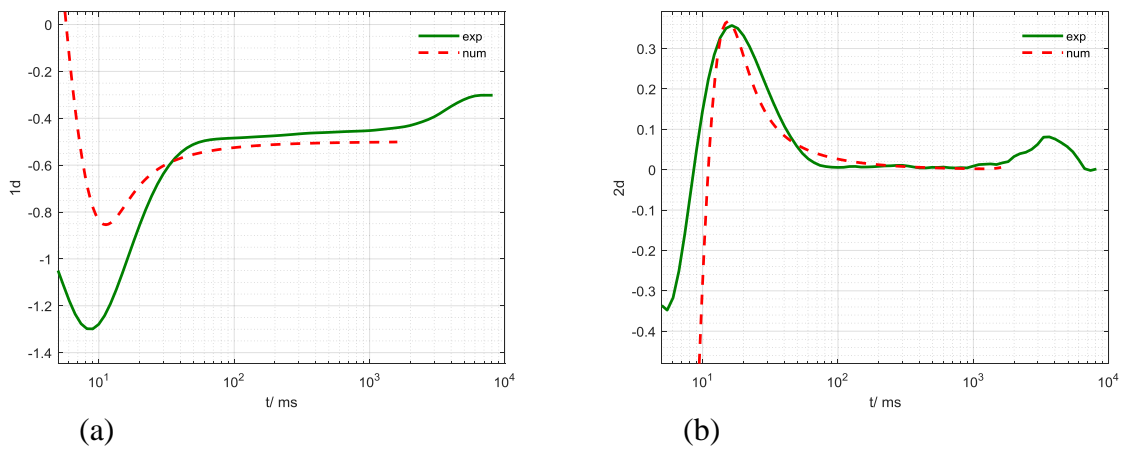

Figure 7: Comparison of flash duration effect on a quartz specimen (a) first derivative (b) second derivative

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